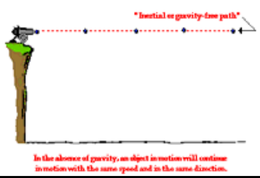
Projectile Motion

Horizontal Motion

- What would happen if we shot a cannon ball at velocity, *v*, horizontally in a world without gravity?
  - It would travel horizontally with constant velocity, *v*

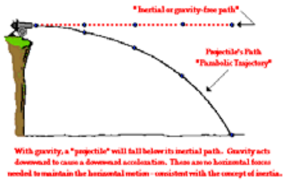


Vertical Motion

- What would happen if we dropped a cannon ball from a cliff?
  - It would fall and accelerate with an acceleration, g

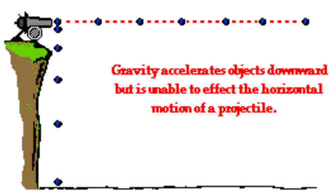
Horizontal and Vertical Motion

- What happens when an object is launched horizontally with gravity?
  - The object follows a parabolic path starting from its launching point and eventually ending on the ground



So what does this mean?

- Since gravity only occurs in the vertical direction, it can only affect the vertical motion



- This means that we will treat the horizontal and vertical components of velocity separately

	Horizontal	**Vertical**
Acceleration	No	Yes g, down ($-9.81\ ms^{-2}$)
Velocity	Constant	Changing

Example

- A cannon ball is launched with a horizontal velocity of 50 ms^{-1} from the top of a 10 m high cliff. Determine the distance from the bottom of the cliff where the cannon ball lands.

We will treat this situation as two separate problems: a horizontal one and a vertical one.

- Horizontal
- $u_x = 50\ ms^{-1}$
- $a = 0$
- $s_x = ?$
- $t = ?$

- Vertical
- $u_y = 0$
- $a = g = -9.81\ ms^{-2}$
- $s_y = -10$ m
- $t = ?$

We have enough information to solve for time, t, vertically.

- Vertical

$$s = ut + \tfrac{1}{2}at^2$$

$$t = \sqrt{\frac{2s_y}{g}}$$

$$t = \sqrt{\frac{2(-10\text{ m})}{-9.81\text{ ms}^{-2}}} = 1.43\text{ s}$$

- The time it takes for the object to fall and hit the ground is the same as the horizontal time
  - The object stops moving horizontally once the object has hit the ground
- That means that we can now solve for the horizontal distance

- Horizontal

$$v = \frac{s}{t}$$

$$s_x = u_x t$$

$$s_x = (50\ \mathrm{ms}^{-1})(1.43\ \mathrm{s}) = 71.5\ \mathrm{m}$$

- But what if the object is launched at an angle?
  - No problem, we treat it exactly the same way

Example

- A cannon ball is launched with a velocity of 50 ms^{-1} at an angle of 30° from the horizontal from the top of a 10 m high cliff. Determine the distance from the bottom of the cliff where the cannon ball lands.

- Once again, we need to separate the horizontal and vertical components
- This time, however, the initial velocity is a vector at an angle
- That means that we have a velocity in both the horizontal and vertical directions

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So, let's write down what we know

- Horizontal
- $u_x = 50\cos 30\ ms^{-1}$
- a = 0
- $s_x = ?$
- t = ?

- Vertical
- $u_y = 50\sin 30\ ms^{-1}$
- $a = -g = -9.81\ ms^{-2}$
- $s_y = -10$ m
- t = ?

Once again, we have enough information to solve for time, t, vertically.

- vertical

$$s = ut + \tfrac{1}{2}at^2$$

$$-10\ \text{m} = (50\sin 30\ \text{ms}^{-1})t + \tfrac{1}{2}(-9.81\ \text{ms}^{-2})t^2$$

$$4.905t^2 - 25t - 10 = 0$$

- We have to solve this using the quadratic formula

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{-(-25) \pm \sqrt{(-25)^2 - 4(4.905)(-10)}}{2(4.905)}$$

$$t = \begin{cases} -0.37\ \text{s} \\ 5.47\ \text{s} \end{cases}$$

- Since time cannot be negative, the only value that makes sense is 5.47 s
- Once again, the horizontal part takes the same amount of time
- So now we can solve the horizontal part

- Horizontal

$$v = \frac{s}{t}$$

$$s_x = u_x t$$

$$s_x = (50\cos 30\ \text{ms}^{-1})(5.47\ \text{s})$$

$$s_x = (43.3\ \text{ms}^{-1})(5.47\ \text{s}) = 237\ \text{m}$$

- We will use this technique to solve all projectile motion problems